



A Novel Method for Determining Thermal Parameters of Moist Material

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Abstract. The study conducted the research on establishing mathematical model for drying process, in which the temperature field and moisture content were also determined. Moreover, the thermal conductivity equation, single-valued conditions, boundary conditions and thermal physics parameters of moist materials were determined. In this study, a novel method was proposed to simultaneously determine the thermal conductivity coefficient, thermal diffusion coefficient and specific heat capacity of moist materials by numerical analysis, and based on approximate roots of the thermal conductivity equation with symmetric boundary conditions of second type when Fourier was small enough. The formulae were established for determining the thermal conductivity coefficient, thermal diffusion coefficient and specific heat capacity of moist materials. Based on this result, the drying experiments were conducted using rice ($\omega = 12\%$, $t = 29^\circ\text{C}$) and pollen ($\omega = 30\%$, $t = 21^\circ\text{C}$) as the materials to detect the error and evaluate the reliability of the method we proposed. In comparison to the results of the previous study, the method error for thermal conductivity coefficient of rice was 7.8%, and for thermal diffusion coefficient was 7.7%, the method error for thermal conductivity coefficient of pollen was 1.05%, and for thermal diffusion coefficient was 4.9%. The comparison results showed that the error of the method proposed was acceptable, which was helpful for the dryer design.

Keywords: Heat conduction · Boundary condition · Thermal conductivity coefficient · Thermal diffusion coefficient · Specific heat capacity

1 Introduction

In drying process, thermal properties of drying material such as thermal conductivity coefficient, thermal diffusion coefficient and specific heat capacity are really important. They are continuously affected by the physical properties of the material such as: moisture content, density, structure during drying process. Thermal properties are used for modeling drying processes and engineering design of processing equipment [1–3].

There were numerous studies of the methods of determining the thermal parameters of materials. Muramatsu et al. [4] developed a new non-isothermal and nonlinear determination method of thermal diffusivity and to measure the thermal diffusivity of low-moisture foods using an inverse technique. The five kinds of low-moisture foods were used including almond meal, corn meal, wheat flour, chocolate fudge, and peanut butter. The one-dimensional transient heat conduction problem for radial coordinates was solved with a finite-difference model. The results showed that the predicted and observed temperature matched well and the thermal diffusivity values of the samples ranged from 9.8×10^{-8} to 1.3×10^{-7} m²/s. Moreover, the estimation method is simple, rapid, and can be conducted for a large spatial temperature gradients. Zhang et al. [5] measured the in-plane thermal conductivity of anisotropic materials using the transient plane source (TPS) method. The results showed that the practical sensor thickness affected the theoretical accuracy of measured anisotropic thermal conductivities. Rastislav et al. [6] determined experimentally the thermal conductivity, thermal diffusivity, specific heat capacity of oriented strand boards using non-stationary extended dynamic plane source (EDPS) experimental method. The experimental results was used to establish regression dependency equations between the thermal properties and the moisture content, in which, the determined dependencies showed a high correlation rate. Cinthia et al. [7] conducted the experimental drying of plum slices using vertical tray-drying and freeze-drying methods and the drying models were used to determine the thermal conductivity coefficient and specific heat. The result showed that the two thermal parameters get the maximum decrease of 90 and 91.5% during drying process. Wilton et al. [8] proposed a two-dimensional model to determine thermal diffusivity and convective heat transfer coefficient for a product with cylindrical geometry during its cooling. The model was proposed using a two-dimensional numerical solution of the diffusion equation for the direct problem and an optimizer program based on the Levenberg Marquardt algorithm for the inverse problem. The results allowed establishing a confidence band that can be used to graphically evaluate the precision of a new simulation for a cucumber with different dimensions. Karistak et al. [9] determined the coefficients of thermal conductivity, thermal diffusivity, and specific heat capacity of solid wood of coniferous trees using the EDPS (extended dynamic plane source) method. The results showed that the data gathered using the EDPS method can be accepted in comparison with the previous studies with the already published models to calculate individual parameters. Vedat et al. [10] studied the determination of thermal conductivity properties in some wood species. The results showed that the relation between wood density and thermal conductivity was linear. Quang et al. [11] studied the thermal diffusivity change of *Codonopsis javanica* in drying with the support of the ultrasound using the methods of theoretical calculations, experimental data, and the particle swarm optimization (PSO) algorithm. The results showed that the thermal diffusivity increased with drying air temperature and ultrasound intensity. Rachel et al. [12] conducted the measurement of the thermal conductivity and volumetric heat capacity of tropical wood species using asymmetrical hot plate method. The results showed that increase of moisture content induces a increase of the thermal conductivity and volumetric heat capacity and the value of the thermal conductivity was different in the longitudinal direction, radial and tangential direction, while the value of volumetric heat capacity was not influenced.

In drying technique, due to the process of heat transfer and moisture diffusion occurring during the initially unstable process, the thermal physics parameters in general and the thermal conductivity coefficient, thermal diffusion coefficient in particular of these materials are only determined by the unstable method. With the desire to contribute to science to enrich all our lives, in this study, a new method is proposed, which can simultaneously determine the two coefficients: thermal conductivity coefficient, thermal diffusion coefficient and specific heat capacity of moist materials in general and pollen in particular at a certain initial temperature and moisture content.

The mathematical basis of the method proposed is based on the two approximate roots of the thermal conductivity equation with symmetric boundary conditions of second type when Fourier is small enough.

2 Materials and Methods

2.1 Physical Model

Let us consider a $2L$ -thick moist flat plate with the known initial moisture content and temperature ω_0 and t_0 , respectively. When $\tau > 0$, both sides of the sheet maintain a constant heat flow density q (W/m^2). Due to the slower moisture diffusion rate compared to the heat transfer rate [13–16] when Fourier is small enough, the average temperature of the flat sheet increases from t_0 to t_{ib} but the average moisture content of the entire sheet hardly changes and is roughly equal to the initial moisture content. Therefore, the thermal conductivity coefficient and thermal diffusion coefficient during this period are equal to the above-mentioned parameters: initial moisture content and average temperature.

In order to determine the temperature field as well as the average temperature of the moist materials in such a brief period, we have N temperature values in the half plate. Then by experiment, we can easily measure the temperature $t_1, t_2, \dots, t_N$; where t_1 is the surface temperature in contact with the heat source (W/m^2), t_N is the temperature at the center of the flat plate. We assume that $t_N = t_0$ and start to increase at the time $\tau = \tau_N$. In other words, when $\tau = \tau_N$, the thermal penetration depth is equal to half the thickness of the flat plate L . Physical model and qualitative temperature distribution in half of the plate are shown in Fig. 1. From this empirical model, we can easily determine the temperature t_1 and the integral average temperature t_{ib} at $\tau = \tau_N$.

2.2 Mathematical Model

Based on the physical model assumed above, according to Crank [17], Tran Van Phu [14, 18], Anabel Alejandra [19], Wei Jin [20] when ignoring the mutual effect between thermal conductivity and moisture diffusion or taking into account, as well as considering the material to be homogeneous and isotropic, and the thermophysical properties not change during thermal conductivity process; the mathematical model for determining the temperature field in the half flat-plate with symmetric boundary conditions of second type is given by Eq. (1):

$$\frac{\partial t(x, \tau)}{\partial \tau} = a \frac{\partial^2 t(x, \tau)}{\partial x^2}, 0 \leq x \leq L, \tau > 0 \quad (1)$$

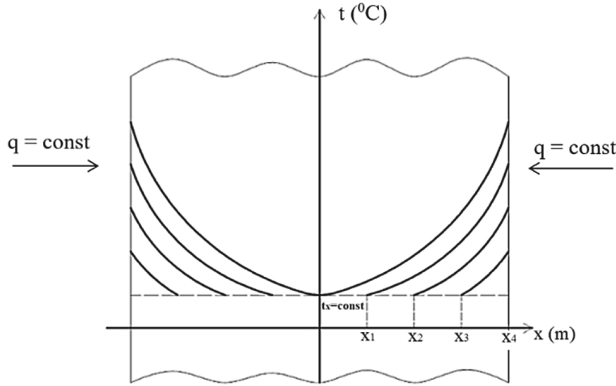


Fig. 1 Spatial temperature profile $t = f(x, \tau)$

Single-valued conditions.

Initial condition is given as Eq. (2):

$$t(x, 0) = t_0 = \text{const} \quad (2)$$

Boundary condition is given as Eq. (3):

$$q = \lambda \left(\frac{\partial t(L, \tau)}{\partial x} \right) \quad (3)$$

Symmetric condition is given as Eq. (4):

$$\left(\frac{\partial t(0, \tau)}{\partial x} \right) = 0 \quad (4)$$

To solve the above problem, we first rewrite Eqs. (1)–(4) in dimensionless form.

Dimensionless coordinate is given by Eq. (5):

$$X = \frac{x}{L} \quad (5)$$

Dimensionless time is given by Eq. (6):

$$Fo = \frac{a\tau}{L^2} \quad (6)$$

Dimensionless temperature is given by Eq. (7):

$$\theta(X, Fo) = \frac{t(x, \tau) - t_0}{t_0} \quad (7)$$

Using the properties of the derivative, we get the differential equation in dimensionless form as follows:

$$\frac{\partial \theta(X, Fo)}{\partial Fo} = \frac{\partial^2 \theta(X, Fo)}{\partial X^2}, 0 \leq X \leq 1, Fo > 0 \quad (8)$$

Initial condition is given by Eq. (9):

$$\theta(X, 0) = 0 \quad (9)$$

Boundary condition is given by Eq. (10):

$$Ki = \frac{\partial \theta(1, Fo)}{\partial X} \quad (10)$$

where $Ki = \frac{qL}{\lambda t_0}$ is the dimensionless heat flow or Kirpychev formula.

Symmetric condition given as Eq. (11):

$$\frac{\partial \theta(0, Fo)}{\partial X} = 0 \quad (11)$$

2.3 Two Roots of the Thermal Conductivity Equation

The unstable one-dimensional thermal conductivity problem with symmetric boundary conditions of second type given by Eq. (8) with single-valued conditions in Eqs. (9)–(11) has been solved by Tran Van Phu [14], Luikov [21, 22], Latif [23], Younis [24] as well as several other authors by different methods, including the integral transform method. Nevertheless, the accurate analytical solution they obtained is in the form of series, which is difficult to use in actual calculations.

In order to find the approximate analytical solution as well as to determine the surface temperature and integral average temperature during the early heating period, we solve the problem by using Laplace transform method.

We set:

$$\theta(X, s) = \int_0^\infty \theta(X, Fo) \exp(-sFo) dFo \quad (12)$$

From Eq. (12), $\theta(X, s)$ is in image form of the temperature function $\theta(X, Fo)$, s is a complex number.

Then, we multiply both sides of Eq. (12) by $\exp(-sFo)$ and performing the integration with Fo from 0 to ∞ , we obtain:

$$\int_0^\infty \frac{\partial \theta(X, Fo)}{\partial Fo} \exp(-sFo) dFo = \int_0^\infty \frac{\partial^2 \theta(X, Fo)}{\partial X^2} \exp(-sFo) dFo \quad (13)$$

We find that the right side of Eq. (13) integrates with time variable Fo but derives by the space variable X . Therefore, we put the derivative sign outside the integral. Then, the right side of Eq. (13) is rewritten as:

$$\int_0^\infty \frac{\partial^2 \theta(X, Fo)}{\partial X^2} \exp(-sFo) dFo = \frac{\partial^2}{\partial X^2} \int_0^\infty \theta(X, Fo) \exp(-sFo) dFo \quad (14)$$

We note that image $T(X, s)$ is the function of variable X when compare Eqs. (12) and (14), so the quadratic partial derivative of X turns into the normal derivative of the image $T(X, s)$. So:

$$\int_0^\infty \frac{\partial^2 \theta(X, Fo)}{\partial X^2} \exp(-sFo) dFo = \frac{d^2 \theta(X, s)}{dX^2} \quad (15)$$

On the other hand, to get the integral of the left side of Eq. (13), we set: $U = \exp(-sFo)$ and $dV = \frac{\partial \theta(X, Fo)}{\partial Fo} dFo$. Therefore, we have: $dU = -s \exp(-sFo) dFo$ and $V = \theta(X, Fo)$. According to partial integration, we have:

$$\begin{aligned} \int_0^\infty \frac{\partial \theta(X, Fo)}{\partial Fo} \exp(-sFo) dFo &= \int_0^\infty U dV = [UV]_0^\infty - \int_0^\infty V dU \\ &= [\exp(-sFo) \theta(X, Fo)]_0^\infty + \int_0^\infty s \theta(X, Fo) \exp(-sFo) dFo \end{aligned}$$

From the initial condition: $\theta(X, 0) = 0$, we have: $[\exp(-sFo) \theta(X, Fo)]_0^\infty = 0$. Therefore

$$\int_0^\infty \frac{\partial \theta(X, Fo)}{\partial Fo} \exp(-sFo) dFo = s \int_0^\infty \theta(X, Fo) \exp(-sFo) dFo = s \theta(X, s) \quad (16)$$

From Eqs. (13), (15) and (16), we obtain the differential equation in the image form as follows:

$$\frac{d^2 \theta(X, s)}{dX^2} - s \theta(X, s) = 0 \quad (17)$$

Thus, by using Laplace transform method, we have excluded the time variable Fo from Eq. (8) and obtained ordinary differential equation.

We now convert the single-valued conditions in Eqs. (9)–(11) into image form. We multiply $\exp(-sFo)$ to Eqs. (10) and (11) and performing the integration with Fo from 0 to ∞ :

$$-\int_0^\infty \frac{\partial T(1, Fo)}{\partial X} \exp(-sFo) dFo + Ki \int_0^\infty \exp(-sFo) dFo = 0 \quad (18)$$

Again, the first term of Eq. (18) integrates with time variable but derives by the space variable X . Therefore, we put the derivative sign outside the integral, we have:

$$-\frac{d}{dX} \int_0^\infty \theta(1, Fo) \exp(-sFo) dFo = -\frac{d\theta(1, s)}{dX} \quad (19)$$

On the other hand, the second term of Eq. (18) is equal to:

$$Ki \int_0^\infty \exp(-sFo) dFo = -Ki \frac{\exp(-sFo)}{s} \Big|_0^\infty = \frac{Ki}{s}. \quad (20)$$

From Eqs. (18)–(20), we obtain the boundary condition with equation:

$$-\frac{d\theta(1, s)}{dX} + \frac{Ki}{s} = 0 \quad (21)$$

We also do the same procedure for symmetric condition:

$$\int_0^\infty \frac{\partial \theta(0, Fo)}{\partial X} \exp(-sFo) dFo = \frac{\partial}{\partial X} \int_0^\infty \theta(0, Fo) \exp(-sFo) dFo = \frac{d\theta(0, s)}{dX} = 0 \quad (22)$$

We need to solve the Eq. (20) with the conditions (21) and (22).

Equation (17) is the usual second derivative equation. According to Luikov [21, 22], Quang et al. [11]. The general solution of the equation has the form:

$$\theta(X, s) = A \operatorname{ch} \sqrt{s} X + B \operatorname{sh} \sqrt{s} X \quad (23)$$

In order to find A and B in Eq. (23), we use boundary condition and symmetric condition Eq. (21) and Eq. (22). We derive Eq. (24):

$$\frac{d\theta(X, s)}{dX} = A \sqrt{s} \operatorname{sch} \sqrt{s} X + B \sqrt{s} \operatorname{ssh} \sqrt{s} X \quad (24)$$

Substituting $X = 0$ and $X = 1$ into Eq. (24), we obtain from Eqs. (21), (22) the following system of equations:

$$\frac{dT(0, s)}{dX} = A \sqrt{s} \operatorname{sch} \sqrt{s} \cdot 0 + B \sqrt{s} \operatorname{ssh} \sqrt{s} \cdot 0 = 0 \quad (25)$$

$$\frac{dT(1, s)}{dX} = A \sqrt{s} \operatorname{sch} \sqrt{s} \cdot 1 + B \sqrt{s} \operatorname{ssh} \sqrt{s} \cdot 1 = \frac{Ki}{s} \quad (26)$$

According to the definition of hyperbolic function:

$$\operatorname{ch} \sqrt{s} X = \frac{\exp(\sqrt{s} X) + \exp(-\sqrt{s} X)}{2} \quad (27)$$

$$\operatorname{sh} \sqrt{s} X = \frac{\exp(\sqrt{s} X) - \exp(-\sqrt{s} X)}{2} \quad (28)$$

Indeed, substituting $X = 0$ into Eqs. (27) and (28), we have:

$$\operatorname{sh} \sqrt{s} \cdot 0 = \frac{\exp(\sqrt{s} \cdot 0) - \exp(-\sqrt{s} \cdot 0)}{2} = 0 \quad (29)$$

$$\operatorname{ch} \sqrt{s} \cdot 0 = \frac{\exp(\sqrt{s} \cdot 0) + \exp(-\sqrt{s} \cdot 0)}{2} \neq 0 \quad (30)$$

From Eqs. (25), (29) and (30) we have:

$$A \sqrt{s} \operatorname{sch} \sqrt{s} \cdot 0 + B \cdot 0 = 0 \quad (31)$$

On the other hand, $\operatorname{ch} \sqrt{s} \cdot 0 \neq 0$.

Then we have the constant of integration: $A = 0$.

Substituting $A = 0$ into Eq. (26), we have: $B = \frac{Ki}{s \sqrt{s} \operatorname{ssh} \sqrt{s}}$.

Thus, the partial solution of the image function has the form:

$$\theta(X, s) = \frac{Ki \operatorname{ch} \sqrt{s} X}{s \sqrt{s} \operatorname{ssh} \sqrt{s}} \quad (32)$$

Expansion of $\text{ch}\sqrt{s}X$ and $\text{sh}\sqrt{s}$ in series, using Laplace transform table and expansion theorem Luikov [21, 22], Samet Ozturk et al. [25], we find the accurate analytical solution:

$$\theta(X, Fo) = Ki \left[Fo - \frac{1}{6}(1 - 3X^2) + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{\mu_n^2} \cos \mu_n X \exp(-\mu_n^2 Fo) \right] \quad (33)$$

where $\mu_n = n\pi$; $n = (1 \div \infty)$.

Clearly, the accurate analytical solution of Eq. (33) is difficult to use in actual calculations. Thus, we find an approximate analytical solution in the early heating period. Mathematically, it is said that finding the approximate analytic solution when Fo is small enough. Then we do not expand in series as when determining the accurate analytical solution, but rather using definition:

$$\text{ch}\sqrt{s}X = \frac{\exp(\sqrt{s}X) + \exp(-\sqrt{s}X)}{2} \quad (34)$$

On the other hand, $\frac{1}{\text{sh}\sqrt{s}}$ can be represented in series:

$$\frac{1}{\text{sh}\sqrt{s}} = 2[\exp(-\sqrt{s}) + \exp(-3\sqrt{s}) + \dots + \exp(-(2n-1)\sqrt{s})] \quad (35)$$

Then, Eq. (32) is equal to:

$$\theta(X, s) = \frac{Ki}{s\sqrt{s}} \{ [\exp\sqrt{s}X + \exp(-\sqrt{s}X)] [\exp(-X) + \exp(-3X) + \dots + \exp(-(2n-1)X)] \} \quad (36)$$

Equation (36) can be rewritten as follows:

$$\theta(X, s) = \frac{Ki}{s\sqrt{s}} \sum_{n=1}^{\infty} \{ \exp(-\sqrt{s}[(2n-1) + X]) + \exp(-\sqrt{s}[(2n-1) - X]) \} \quad (37)$$

Using Laplace table, we find from Eq. (37) the solution in the following form when Fo is small enough:

$$\theta(X, Fo) = 2Ki\sqrt{Fo} \sum_{n=1}^{\infty} \left[\text{ierfc} \frac{(2n-1) - X}{2\sqrt{Fo}} + \text{ierfc} \frac{(2n-1) + X}{2\sqrt{Fo}} \right] \quad (38)$$

According to the Laplace table, we have:

$$\frac{1}{s\sqrt{s}} \sum_{n=1}^{\infty} \exp(-\sqrt{s}[(2n-1) - X]) = 2\sqrt{Fo} \sum_{n=1}^{\infty} \text{ierfc} \frac{(2n-1) - X}{2\sqrt{Fo}} \quad (39)$$

$$\frac{1}{s\sqrt{s}} \sum_{n=1}^{\infty} \exp(-\sqrt{s}[(2n-1) + X]) = 2\sqrt{Fo} \sum_{n=1}^{\infty} \text{ierfc} \frac{(2n-1) + X}{2\sqrt{Fo}} \quad (40)$$

or

$$\theta(X, Fo) = 2Ki\sqrt{Fo} \sum_{n=1}^{\infty} \text{ierfc} \frac{(2n-1) \pm X}{2\sqrt{Fo}} \quad (41)$$

Solution of Eq. (41) is another form of the accurate analytical solution to the problem, which we are considering when Fo is small enough. According to Luikov [21, 22], Xiao-Jun Yang [26], when Fo is small enough then $n = 1$. Then, the solution Eq. (41) is approximately equal to:

$$\theta(X, Fo) \approx 2Ki\sqrt{Fo} \left\{ \operatorname{ierfc} \frac{1 \pm X}{2\sqrt{Fo}} \right\} \quad (42)$$

In Eq. (41), $\operatorname{ierfc} \frac{(2n-1)\pm X}{2\sqrt{Fo}}$ is a special function which is given in a tabular form according to Luikov [21, 22], Crank [17], Borui Li [27].

Equation (42) is used to calculate the surface temperature of the flat plate $X = 1$, when proposing method for simultaneously determining the thermal conductivity coefficient and thermal diffusion coefficient.

On the other hand, according to Tran Van Phu [18], Zhukovsky [28], Julien Berger [29], provided that Fo is small enough or $n = 1$, the solution of Eq. (37) has the form:

$$\theta(X, s) = \frac{Ki}{s\sqrt{s}} \left\{ \exp[-\sqrt{s}(1+X)] + \exp[-\sqrt{s}(1-X)] \right\} \quad (43)$$

or

$$\theta(X, s) = \frac{Ki}{s\sqrt{s}} \left\{ \exp[-\sqrt{s}(1 \pm X)] \right\} \quad (44)$$

In this method, we separate the image $\theta(X, s)$ in Eq. (44) into two functions for s . For that purpose, we set: $F_1(s) = \frac{Ki}{s}$ and,

$$F_2(X, s) = \frac{1}{\sqrt{s}} \exp[-\sqrt{s}(1 \pm X)] \quad (45)$$

Then, solution of Eq. (44) is expressed as the product of $F_1(s)$ and $F_2(X, s)$:

$$\theta(X, s) = F_1(s)F_2(X, s) = \frac{Ki}{s\sqrt{s}} \left\{ \exp[-\sqrt{s}(1 \pm X)] \right\} \quad (46)$$

According to the Laplace transform, if the original function of $F_1(s)$ is $f_1(Fo)$ and the original function of $F_2(X, s)$ is $f_2(X, Fo)$ then the original function of the image function $\theta(X, s) = F_1(s)F_2(X, s)$ is:

$$\theta(X, Fo) = f_1(Fo) * f_2(X, Fo) = \int_0^{Fo} f_1(Fo - \theta) f_2(X, \theta) d\theta \quad (47)$$

In Eq. (47), the symbol $*$ is convolution product (variable Fo) of the two functions $f_1(Fo)$ and $f_2(X, Fo)$.

According to the Laplace table [6], we can easily find the original function of the image functions $F_1(s)$ and $F_2(X, s)$. For image, its original function is: $f_1(Fo - \theta) = Ki = \text{const.}$

Then the image function $F_2(X, s) = \frac{1}{\sqrt{s}} \exp[-\sqrt{s}(1 \pm X)]$ has the original function:

$$f_2(X, Fo) = \frac{1}{\sqrt{\pi Fo}} \exp\left[-\frac{(1 \pm X)^2}{4Fo}\right].$$

From Eq. (47), we have:

$$\theta(X, Fo) = Ki \int_0^{Fo} \frac{1}{\sqrt{\pi Fo}} \exp\left[-\frac{(1 \pm X)^2}{4Fo}\right] dFo \quad (48)$$

$$\theta(X, Fo) = Ki \int_0^{Fo} \frac{2}{\sqrt{4\pi Fo}} \exp\left[-\frac{(1 \pm X)^2}{4Fo}\right] dFo$$

or

$$\theta(X, Fo) = 2Ki \int_0^{Fo} \frac{1}{\sqrt{4\pi Fo}} \exp\left[-\frac{(1 \pm X)^2}{4Fo}\right] dFo \quad (49)$$

We now use Eq. (49) to calculate the average value of integral at $\tau = \tau_n$.

2.4 Constructing Formula for Determining Thermal Conductivity Coefficient, Thermal Diffusion Coefficient and Specific Heat Capacity

As mentioned above, the mathematical basis of our proposed method is the system of equations of two approximate analytical solutions of thermal conductivity equation with symmetric boundary conditions of second type in the one-dimensional half plate when Fourier is small enough. Therefore, we use Eq. (42) to determine the surface temperature in contact with the heat source and Eq. (49) to calculate the average value of integral at the time $\tau = \tau_n$.

Surface temperature of the material ($X = 1$) at the time $\tau = \tau_n$.

As discussed above, we use the approximate analytical solution Eq. (42) to determine the temperature on the surface in contact with the heat source. Then, substituting $X = 1$ into Eq. (48) we have:

$$\theta_1(1, Fo) = 2Ki\sqrt{Fo} \left(ierfc \frac{1}{\sqrt{Fo}} + ierfc0 \right) \quad (50)$$

Due to the properties of function $ierfc$, when Fo is small enough, we can neglect $ierfc \frac{1}{\sqrt{Fo}}$.

For example, when $Fo = 0.5$ then $ierfc \sqrt{1/0.5} = 0.001$. Meanwhile, $ierfc0 = \frac{1}{\sqrt{\pi}} = 0.5642$.

Thus, when $Fo = 0.5$, neglecting $ierfc \frac{1}{\sqrt{Fo}}$ we only make a minor relative error δ which is $\delta = \frac{0.0010}{(0.5642+0.0010)} = 0.0018$.

Thus, when Fo is small enough, we always have $ierfc0 \gg ierfc \frac{1}{\sqrt{Fo}}$. Therefore, the surface temperature of the material which is in contact with the heat source or $X = 1$ is approximately equal to:

$$\theta_1(1, Fo) \approx \frac{2}{\sqrt{\pi}} Ki \sqrt{Fo} \quad (51)$$

Equation (51) is one of two equations of the system, that allows us to simultaneously determine the thermal conductivity coefficient and thermal diffusion coefficient. The second equation of this system is the integral average temperature at τ . Below, we show how to obtain this relationship.

The integral average temperature t_{ib} of the flat plate at $\tau = \tau_N$.

To determine the integral average temperature, we perform integration from -1 to 1 in Eq. (49):

$$\theta_{ib}(Fo) = \frac{2Ki}{|1 - (-1)|} \int_0^{Fo} \left\{ \int_{-1}^1 \frac{1}{\sqrt{4\pi Fo}} \exp \left[-\frac{(1 \pm X)^2}{4Fo} \right] dX \right\} dFo$$

or

$$\theta_{ib}(Fo) = Ki \int_0^{Fo} \left\{ \int_{-1}^1 \frac{1}{\sqrt{4\pi Fo}} \exp \left[-\frac{(1 \pm X)^2}{4Fo} \right] dX \right\} dFo \quad (52)$$

In Eq. (52), $f_2^*(X, Fo) = \frac{1}{\sqrt{4\pi Fo}} \exp \left[-\frac{(1 \pm X)^2}{4Fo} \right]$ is a very special function that often appears in technical problems describing the initial transition process with Fourier small enough. This function was well researched by Grinber G. and introduced by Luikov [21, 22], Hankun Zhu [30]. Qualitatively, function $f_2^*(X, Fo)$ is shown in Fig. 2.

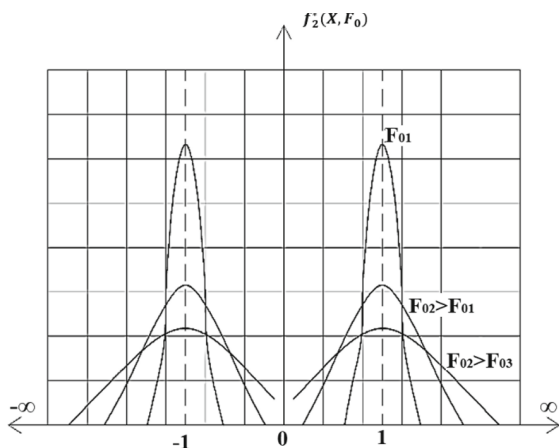


Fig. 2 Functional distribution of $f_2^*(X, Fo) = \frac{1}{\sqrt{4\pi Fo}} \exp \left[-\frac{(1 \pm X)^2}{4Fo} \right]$

According to Grinber, the function $f_2^*(X, Fo)$ has the following three properties:

- (i) two symmetric curves about two axes: $X = 1$ and $X = -1$;
- (ii) the smaller the Fo , the more concentrated around two axes the area limited by $f_2^*(X, Fo)$;
- (iii) when integrating by X from $-\infty$ to $+\infty$, we always have the following relationship:

$$\int_{-\infty}^{+\infty} \frac{1}{\sqrt{4\pi Fo}} \exp\left[-\frac{(1+X)^2}{4Fo}\right] dX = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\pi 4Fo}} \exp\left[-\frac{(1-X)^2}{4Fo}\right] dX = 1 \quad (53)$$

Thus:

$$\begin{aligned} \int_{-\infty}^{+\infty} f_2^*(X, Fo) dX &= \int_{-\infty}^{+\infty} \frac{1}{\sqrt{4\pi Fo}} \exp\left[-\frac{(1+X)^2}{4Fo}\right] dX \\ &+ \int_{-\infty}^{+\infty} \frac{1}{\sqrt{4\pi Fo}} \exp\left[-\frac{(1-X)^2}{4Fo}\right] dX = 2 \end{aligned} \quad (54)$$

Due to the second property for $f_2^*(X, Fo)$, when Fo is small enough as shown in Fig. 2, we have:

$$\int_{-1}^{+1} \frac{1}{\sqrt{4\pi Fo}} \exp\left[-\frac{(1+X)^2}{4Fo}\right] dX + \int_{-1}^{+1} \frac{1}{\sqrt{4\pi Fo}} \exp\left[-\frac{(1-X)^2}{4Fo}\right] dX \approx \frac{1}{2} + \frac{1}{2} \approx 1. \quad (55)$$

We now take advantage of this special property of the function $f_2^*(X, Fo)$ shown by Eq. (55) to calculate the integral average temperature in the half plate.

Then, substituting Eqs. (55) into (52), the integral average temperature at $\tau = \tau_n$ is approximately equal to:

$$\theta_{tb}(Fo) \approx Ki \int_0^{Fo} dFo \approx Ki \cdot Fo \quad (56)$$

From Eqs. (51) and (56), we obtain the system of equations that allows us to simultaneously determine the thermal conductivity coefficient and thermal diffusion coefficient.

Thermal conductivity coefficient (λ) and thermal diffusion coefficient (a) are the two variables of the following system of equations:

$$\begin{cases} \theta_1(1, Fo) = \frac{2}{\sqrt{\pi}} Ki \sqrt{Fo} \\ \theta_{tb}(Fo) = Ki \cdot Fo \end{cases} \quad (57)$$

Substituting $Fo = \frac{a\tau}{L^2}$, $Ki = \frac{qL}{\lambda t_0}$ and $\theta(X, Fo) = \frac{t(x, \tau) - t_0}{t_0}$ into Eq. (57), we obtain the system of equations for the thermal conductivity coefficient (λ) and thermal diffusion coefficient (a) as follows:

$$\begin{cases} \frac{t_1(\tau) - t_0}{t_0} = \frac{2qL}{\lambda t_0 \sqrt{\pi}} \sqrt{\frac{a\tau}{L^2}} \\ \frac{t_{tb}(\tau) - t_0}{t_0} = \frac{qLa\tau}{\lambda t_0 L^2} \end{cases} \quad (58)$$

Solving the system of Eq. (58), we find two formulae that allow us to simultaneously determine these two coefficients.

Thermal conductivity coefficient:

$$\lambda = \frac{4qL[t_{tb}(\tau) - t_0]}{\pi[t_1(\tau) - t_0]^2} \quad (59)$$

Thermal diffusion coefficient:

$$a = \frac{4L^2[t_{tb}(\tau) - t_0]^2}{\pi\tau[t_1(\tau) - t_0]^2} \quad (60)$$

In order to verify the accuracy of the two Eqs. (59), (60), we introduce the specific heat capacity calculation formula in the above experiment.

Consider the material: $G = FL\rho$

Where FL (m^3) is the volume and G (kg) is the mass of the material.

Then, according to the specific heat capacity definition of material as well as the formulae in [13, 31]. The calorie Q of material mass G needed during the time τ of its temperature rising from t_0 to t_{tb} is calculated as follows:

$$Q = GC\Delta t = GC(t_{tb} - t_0) \quad (61)$$

or

$$C = \frac{Q}{G\Delta t} = \frac{Q}{LF\rho\Delta t} = \frac{q\tau}{L\rho\Delta t} = \frac{q\tau}{L(t_{tb} - t_0)\rho} \quad (62)$$

where q [W/m^2] is the heat-flow density. Therefore, $Q = q\tau$ is the amount of heat needed to raise to temperature of G [kg] material from t_0 to t_{tb} .

On the other hand, if we substitute λ and a from Eqs. (59) and (60) into the formula for determining specific heat capacity: $C = \lambda/(a\rho)$ from the previous studies [21–23], we also obtain:

$$C = \frac{\lambda}{a\rho} = \frac{q\tau_N}{L(t_{tb} - t_0)\rho} \quad (63)$$

As mentioned above, the mathematical basis of our proposed method is based on two approximate roots of the thermal conductivity equation with symmetric boundary conditions of second type when Fourier is small enough. Therefore, we now use Eqs. (59), (60) and (62) to simultaneously determine the thermal conductivity coefficient, thermal diffusion coefficient and specific heat capacity of moist materials.

In general, method errors are very difficult to detect. In order to evaluate the reliability of our proposed method based on Eqs. (59) and (60), we shall compare the values of the parameters tested by experiment with published values of the materials from our proposed method.

2.5 Materials and Experimental Equipment

Materials: In order to detect and estimate the method and laboratory equipment errors, we choose rice with a moisture content of $\omega = 12\%$, $t = 29^\circ\text{C}$ and pollen with a moisture content of $\omega = 30\%$, $t = 21^\circ\text{C}$ as reference materials.

Laboratory equipment: as mentioned above, the mathematical basis of the method we propose is based on the two approximate roots of the thermal conductivity equation in one-dimensional flat plate with symmetric boundary conditions of second type when Fourier is small enough. Due to the symmetric property, to determine the temperature field as well as the average temperature of the moist material in such a short time, the half plate is considered. Therefore, we place in half flat plate N equal-spaced heat detectors to measure the temperature $t_1, t_2, \dots, t_N$. Where t_1 is the surface temperature in contact with the heat source q (W/m^2), t_N is the temperature at the center of the flat plate.

To empirically demonstrate the one-dimensional unstable thermal conductivity problem through a half-plate with symmetric boundary conditions of second type, we place a heat source in the middle of the material with a sufficiently large thickness. The heat flow will then travel in two directions, and the heat-flow density through the half plate is equal to $q/2$.

The experimental equipment system model is given in Fig. 3. The experimental equipment system consists of a container (thermal insulation) filled with the research material. In the middle of the container, we place a heat source and assume that the heat flow travels in the horizontal direction. We place five equal-spaced heat detectors connected to the recorder. The first thermocouple is in contact with the heat source (Fig. 3).

Container: inner size $L \times W \times H = 400 \times 150 \times 150 \text{ mm}^3$. It is made of polyurethane (100 mm thickness), coated a layer of tole (0.5 mm thickness) and power coating to ensure heat transfer is one-dimensional (horizontal).

Heat detectors: are Pt100 (platinum resistance thermometers), with a diameter of 2 mm, alength of 150 mm, an accuracy of $\pm 0.1\%$. Heat detectors are equally-spaced placed with a distance of 12.5 mm, the first one in contact with the heat source. The heat detectors are connected to the TZN4L-24R automatic recording device with an accuracy of $\pm 0.3\%$.

Heat source: a resistor with dimensions $L \times W \times H = 150 \times 3 \times 150 \text{ mm}^3$. The heat source is composed of 3 layers pressed tightly together, the outermost layer is a 304 stainless steel of 0.8 mm thickness followed by the heat-resistant cloth layer and the inside is the resistance wire of 0.5 mm diameter, power 50 W. The heating capacity of the heat source is recorded by a self-recording device (Watt meter) BX11 - Trotec (Germany) with the range of 2–3000 W and adjusted by an autotransformer.

3 Results and Discussion

3.1 Results

In order to detect and estimate the method and laboratory equipment errors, we use the comparison method. In general, we use this experimental equipment to determine the thermal conductivity coefficient and thermal diffusion coefficient of a material, which has been published. In this research, we choose rice.

The values of thermal conductivity coefficient and thermal diffusion coefficient of rice are determined by the experimental equipment based on our proposed method with Eqs. (59), (60) are: $\lambda = 0.1132 \text{ W/(m}\cdot\text{K)}$; $a = 1.53 \times 10^{-7} \text{ m}^2/\text{s}$ (see Table 1).

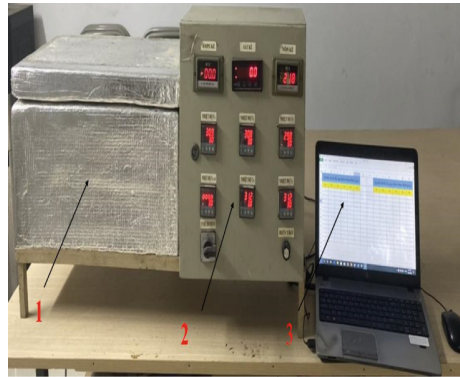


Fig. 3 Experimental equipment for simultaneous determining the thermal conductivity coefficient, thermal diffusion coefficient and specific heat capacity of moist material

Table 1 Thermal conductivity coefficient, thermal diffusion coefficient of rice with moisture content $\omega = 12\%$, temperature $t = 29^\circ\text{C}$

Time	t_1 ($^\circ\text{C}$)	t_2 ($^\circ\text{C}$)	t_3 ($^\circ\text{C}$)	t_4 ($^\circ\text{C}$)	t_5 ($^\circ\text{C}$)	λ (W/m·K)	a (m^2/s)
0	29.0	29.0	29.0	29.0	29.0	0.1132	1.53×10^{-7}
2 min 20 s	33.5	29.1	29.0	29.0	29.0		
5 min 20 s	39.4	29.8	29.1	29.0	29.0		
10 min 5 s	52.9	32.2	28.8	29.1	29.0		
21 min 5 s	67.0	38.0	33.2	30.2	29.1		

Meanwhile, according to Shafiur Rahman [32], the thermal conductivity coefficient, thermal diffusion coefficient of rice is: $\lambda = 0.1050 \text{ W}/(\text{m}\cdot\text{K})$, $a = 1.42 \times 10^{-7} \text{ m}^2/\text{s}$.

Error of our method (for thermal conductivity coefficient):

$$\delta = \left| \frac{0.1132 - 0.1050}{0.1050} \right| \times 100 = 7.8\%$$

Error of our method (for thermal diffusion coefficient):

$$\delta = \left| \frac{1.53 \times 10^{-7} - 1.42 \times 10^{-7}}{1.42 \times 10^{-7}} \right| \times 100 = 7.7\%$$

In addition, the values of thermal conductivity coefficient and thermal diffusion coefficient of pollen are determined by the experimental equipment based on our proposed method with Eqs. (59) and (60) are: $\lambda = 0.1253 \text{ W}/(\text{m}\cdot\text{K})$; $a = 1.4106 \times 10^{-7} \text{ m}^2/\text{s}$, given in Table 2.

Meanwhile, according to Monika Bozikova [33], the thermal conductivity coefficient, thermal diffusion coefficient of pollen is: $\lambda = 0.124 \text{ W}/(\text{m}\cdot\text{K})$; $a = 1.48 \times 10^{-7} \text{ m}^2/\text{s}$.

Error of our method (for thermal conductivity coefficient):

$$\delta = \left| \frac{0.1253 - 0.1240}{0.1240} \right| \times 100 = 1.05\%$$

Error of our method (for thermal diffusion coefficient):

$$\delta = \left| \frac{1.4800 \times 10^{-7} - 1.4106 \times 10^{-7}}{1.4106 \times 10^{-7}} \right| \times 100 = 4.9\%$$

Table 2 Thermal conductivity coefficient, thermal diffusion coefficient of pollen with moisture content $\omega = 30\%$, temperature $t = 21^\circ\text{C}$

Time	t_1 ($^\circ\text{C}$)	t_2 ($^\circ\text{C}$)	t_3 ($^\circ\text{C}$)	t_4 ($^\circ\text{C}$)	t_5 ($^\circ\text{C}$)	λ (W/m·K)	a (m^2/s)
0	21	21	21	21	21	0.1253	1.4106×10^{-7}
2 min 20 s	23.1	21.1	21	21	21		
8 min 20 s	28.6	22.8	21.1	21	21		
15 min 25 s	37.8	24.1	22.3	21.1	21		
26 min 15 s	52.5	28.1	23.8	22.8	21.1		

3.2 Discussion

We notice that Eq. (62) is the equation for determining specific heat capacity based on definition, whereas, Eq. (63) determines specific heat capacity by thermal conductivity coefficient (λ) and thermal diffusion coefficient (a) based on our method. Mathematically, this indirectly proves that the novel method proposed for simultaneous determining the thermal conductivity coefficient in Eq. (59) and thermal diffusion coefficient in Eq. (60) are correct and reliable.

The novel method for simultaneous determining the thermal conductivity coefficient, thermal diffusion coefficient and specific heat capacity that is proposed in Eq. (59), (60) and (62), in our opinion, has the following advantages:

It allows to simultaneously determine the above three coefficients of all moist materials in general and pollen in particular, during the initial short period of unstable thermal conduction. Thus, it helps saving time of doing experiments.

By changing the initial moisture content and temperature, it is possible to establish the relationship between the above coefficients: $\lambda = f(t, \omega)$, $a = f(t, \omega)$ and $C = f(t, \omega)$.

In addition to the above advantages, the formulae also have disadvantages. These formulae were obtained on the basis of two analytical solutions, the method has its own methodological error.

From the above experimental results, we can see that the errors are about 8 and 5% for rice and pollen, respectively. In our opinion, the method is relative comparison, so this percent error is acceptable. This error is due to the following reasons such as: (i)

error in the method of determining the coefficients due to approximate solutions of the method; (ii) error in manufacturing laboratory equipment; (iii) error in operating and measuring process; (iv) error of measuring instruments.

4 Conclusions

Based on theoretical research of moist materials and experiment, we devised formulae for simultaneous determining the thermal conductivity coefficient, thermal diffusion coefficient and specific heat capacity of moist materials.

Thermal conductivity coefficient:

$$\lambda = \frac{4qL(t_{tb}(\tau) - t_0)}{\pi(t_1(\tau) - t_0)^2}, \text{ W/(m} \cdot \text{K)}$$

Thermal diffusion coefficient:

$$a = \frac{4L^2(t_{tb}(\tau) - t_0)^2}{\pi\tau(t_1(\tau) - t_0)^2}, \text{ m}^2/\text{s}$$

Specific heat capacity:

$$C = \frac{q\tau}{L(t_{tb} - t_0)\rho}, \text{ J/(kg} \cdot \text{K)}$$

In comparison to the results of the previous studies, the comparison results showed that the error of the method proposed to determine the thermal parameters of moist materials is acceptable.

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